Ponderomotive focusing in axisymmetric rf linacs

S. C. Hartman and J. B. Rosenzweig
Department of Physics, The University of California at Los Angeles, 405 Hilgard Avenue, Los Angeles, California 90024
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In many present and planned compact electron sources and linacs, very large rf accelerating fields are present in the linac structures. A charged particle whose trajectory is parallel to the axis receives no net focusing from the periodic transverse forces arising from the axisymmetric rf wave. However, if the lowest-order periodic transverse motion of the particles is included, a net *ponderomotive* focusing force is obtained when the momentum transfer is averaged over a period of the motion. Since this force is second order in the electric-field amplitude, the resultant focusing can be non-negligible in high gradient structures. In order to produce and preserve high-brightness beams in rf linacs it is necessary to obtain a better understanding of all the forces acting on the beam and the subsequent beam dynamics. To this end we examine the lowest-order ponderomotive rf focusing of electrons in axisymmetric standing- and traveling-wave linac structures. We obtain a generalized paraxial envelope equation which includes the effects of the ponderomotive focusing and longitudinal acceleration to describe the transverse beam envelope evolution. The implications of our results on beam dynamics in electron linacs are examined, and extensions of the results to positron and proton linacs are discussed.

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I. INTRODUCTION AND OVERVIEW

The focusing of an electron beam in a radio-frequency linear accelerator (rf linac) is derived from the periodic transverse motion of the electrons induced by the transverse Lorentz force on the electrons. Since this force has a transverse spatial gradient, the net momentum transfer integrated over a cycle of the motion is nonzero. This effect is termed a *ponderomotive force,* and is of second order in the field amplitude [1]. All alternating focusing schemes in the smooth approximation (e.g., radio-frequency quadrupole (RFQ) focusing [2]) are based on this effect. Because of this nonlinear dependence of the focusing strength on the field amplitude, this effect can become important rather quickly as the acceleration gradient is increased.

We begin the analysis by deriving the ponderomotive force due to an arbitrary transverse periodic force with a gradient linear in the dimension of the oscillatory motion. This is accomplished by writing the force as a sum of Fourier harmonics, and solving for the motion of the particle initially by ignoring the effect of the gradient. The resulting trajectory of the particle then provides a path over which to evaluate the force, in order to average this quantity over one cycle of the periodic motion. Because of the orthogonality of the Fourier series components, the ponderomotive force can then be written as a simple series in the harmonic number.

The ponderomotive focusing force on a beam in a cylindrically symmetric rf linac cavity operated in an axisymmetric (azimuthal mode number $m=0$) TM mode arises from a periodic transverse Lorentz force, associated with the nonsynchronous field components, those modes which have a phase velocity ω/k not equal to the beam velocity $v_b=\beta_b c$ (where β_b is very near to unity). This force is approximately linear in transverse coordinate r for particle displacements from the axis which are small compared to the cavity radius. The transverse Lorentz force due to the rf wave is derived from the form of the longitudinal field on axis by considering the Fourier series representation of this field. This analysis is initially performed for a standing-wave linac structure; an extension to the traveling-wave case is then presented.

From this, a total effective average focusing strength of the ponderomotive force for an arbitrary $m=0$ TM mode is obtained, which can be used in a generalized envelope equation for the rms beam size. This equation also includes the effects on the beam particle trajectories due to the longitudinal acceleration provided by the rf wave. The envelope equation is then numerically integrated for an example of interest, and the focusing characteristics examined. The implications of these results on linear accelerator design are then discussed, not only for electrons, but for positrons and protons as well.

II. THE PONDEROMOTIVE FORCE: GENERAL CONSIDERATIONS

We now examine the ponderomotive force due to a periodic force which has a constant gradient parallel to the direction of the force vector. Taking this direction to be x, we write the force as a complex Fourier series,

$$F_x = x \sum_{n=-\infty}^{\infty} A_n \exp(in\omega t)\ , \quad A_{-n} = A_n^*\ , A_0 = 0\ . \qquad (1)$$

This compact notation is of course equivalent to a sum of cosine (real) and sine (imaginary) components:

$$F_x = x \sum_{n=1}^{\infty} C_n \cos(n\omega t) + iS_n \sin(n\omega t)\ , \qquad (2)$$

where

$$A_n = \frac{C_n + iS_n}{2}\ , \quad A_n^* = \frac{C_n - iS_n}{2}\ , \qquad (3)$$

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that is, C_n and S_n are real numbers. It is assumed that the force is weak, in the sense that a particle of the mass of interest does not change its position by a large amount over one period $T=2\pi/\omega$. Therefore, for the purposes of deriving the ponderomotive force we solve for the first-order motion, ignoring the x dependence of the force, so that we may perform an average over a period. An equivalent restriction on the validity of this approximation is that the frequency of the applied force ω must be much larger than the frequency of the oscillation derived from the averaged force. For a relativistic particle of energy γmc^2 the equation for the x motion is then, assuming the change in the particle energy over a period is ignorably small, approximately

$$\ddot{x}=\frac{F_x}{\gamma m}=\frac{x_0}{\gamma m}\sum_{n=-\infty}^{\infty} A_n \exp(in\omega t) \ . \tag{4}$$

The homogeneous part of the solution to Eq. (4) is the oscillation about the reference position x_0, and the total displacement in x is written

$$x=x_0\left[1-\frac{1}{\gamma m\omega^2}\sum_{n=-\infty}^{\infty}\left[\frac{A_n}{n^2}\exp(in\omega t)\right]\right] \ . \tag{5}$$

It can be seen by inspection that approximation that $x \simeq x_0$ is valid when ω is large enough or the $A_n/\gamma m$ are small enough. In order to average the motion to obtain the second-order change in momentum over one full period, we substitute the expression for x from Eq. (5) into Eq. (1):

$$\Delta p_x=\int_0^T F_x dt \tag{6}$$

$$=x_0\int_0^T\left[1-\frac{1}{\gamma m_e\omega^2}\sum_{n=-\infty}^{\infty}\frac{A_n}{n^2}\exp(in\omega t)\right] \times \sum_{m=-\infty}^{\infty} A_m \exp(im\omega t)dt \ . \tag{7}$$

The ponderomotive force in this system is thus

$$\overline{F_x}=\frac{\Delta p_x}{T} \tag{8}$$

$$=-\frac{x_0}{\gamma m\omega^2}\sum_{n=-\infty}^{\infty}\frac{|A_n|^2}{n^2} \tag{9}$$

$$=-\frac{2x_0}{\gamma m\omega^2}\sum_{n=1}^{\infty}\frac{|A_n|^2}{n^2}=-\frac{x_0}{2\gamma m\omega^2}\sum_{n=1}^{\infty}\frac{C_n^2+S_n^2}{n^2} \ , \tag{10}$$

a net focusing force for a field which increases linearly in amplitude away from the symmetry axis. The ponderomotive force is focusing in this case because the particle, while oscillating, experiences a slightly larger inward force during the half-cycle it is farther than x_0 from the axis, and a slightly smaller inward force during the opposite half-cycle. Thus there is a net inward, or focusing, impulse imparted to the particle over the full cycle.

III. PONDEROMOTIVE rf FOCUSING IN STANDING-WAVE ACCELERATORS

We now consider a periodic standing-wave π-mode accelerating structure where the electrons have already attained relativistic velocities. The electrons are assumed to be accelerated by the TM axial rf field in the cavities, which is taken to have the following form on axis:

$$E_z(z,t)=E_0\sum_{n=1}^{\infty} a_n\cos(nkz)\sin(\omega t+\phi_0) \ . \tag{11}$$

This assumed periodicity is of sufficient generality to contain all of the physics due to the effects of the modes under investigation, even though we have allowed no arbitrary phases in the arguments of the spatial harmonics. We will only be considering small displacements r from the axis, where the transverse fields are linear, or equivalently the longitudinal field is constant as a function of r. We take a_1 to be normalized to unity, so that E_0 is twice the average accelerating gradient in the limit that $v_b=c$. The focusing force due to this rf wave can be derived by considering the constraints of the Maxwell equations to obtain [3]

$$E_r=-\frac{r}{2}\frac{\partial E_z}{\partial z} \tag{12}$$

and

$$cB_\varphi=\frac{r}{2c}\frac{\partial E_z}{\partial t} \ . \tag{13}$$

The net radial force on an electron is

$$F_r=-e(E_r-v_bB_\varphi) \tag{14}$$

$$=e\left[\frac{r}{2}\right]\left[\frac{\partial}{\partial z}+\beta_b^2\frac{\partial}{\partial z}\right]E_z \ , \tag{15}$$

where we have used

$$\frac{\partial}{\partial z}=\frac{1}{\beta_b c}\frac{\partial}{\partial t} \ . \tag{16}$$

In this way, we reduce the radial force to a single term

$$F_r=e\frac{r}{2}[1+\beta_b^2]\frac{\partial}{\partial z}E_z \simeq er\frac{\partial}{\partial z}E_z \tag{17}$$

for $\beta_b \simeq 1$. For the π-mode structure under consideration one obtains optimal acceleration for the design or *reference* particle when

$$\omega t+\phi_0=kz+\frac{\pi}{2} \ . \tag{18}$$

Since the velocity of the particles after they become relativistic is such that $z=z_0+(\omega/k)t$, i.e., the forward component of the fundamental accelerating wave phase velocity is c, we can write

$$\sin(\omega t+\phi_0)=\sin\left[kz+\frac{\pi}{2}+\Delta\phi\right]=\cos(kz+\Delta\phi) \ , \tag{19}$$

where $\Delta\phi$ is the phase difference between the particle under consideration and the reference particle. We assume that $\Delta\phi$ does not change appreciably as a result of acceleration. This relation allows us to remove the temporal dependence of axial field that the particle encounters, and use z as the independent variable as is customary in accelerator physics:

$$E_z = E_0 \sum_{n=1}^{\infty} a_n \cos(nkz)\cos(kz+\Delta\phi) \ . \tag{20}$$

Note that since the force is linear in r in Eq. (17), one can write its x or y components with the same coefficient as found in Eq. (17). Staying at this point with the radial coordinate for the purpose of analysis, we can derive the Fourier series representation of F_r as follows:

$$F_r = er\frac{\partial E_z}{\partial z} \tag{21}$$

$$= -eE_0 kr \left[\sin(kz+\Delta\phi) \sum_{n=1}^{\infty} a_n \cos(nkz) + n\cos(kz+\Delta\phi) \sum_{n=1}^{\infty} a_n \sin(nkz) \right] . \tag{22}$$

After some manipulation with trigonometric identities, Eq. (22) can be written as

$$F_r = -\frac{eE_0 k}{2} \sum_{n=0}^{\infty} (n+1)\{a_n \sin[(n+1)kz+\Delta\phi] + a_{n+2}\sin[(n+1)kz-\Delta\phi]\} , \quad a_0 = 0 \ . \tag{23}$$

The results of Eq. (10) can be taken over directly, as the derivation does not depend on the choice of independent variables. In this way we obtain the ponderomotive force in terms of the longitudinal field Fourier components,

$$\overline{F_r} = -r\frac{(eE_0)^2}{8\gamma mc^2} \sum_{n=1}^{\infty} \{a_{n-1}^2 + a_{n+1}^2 + 2a_{n-1}a_{n+1}[\cos(2\Delta\phi)]\} , \quad a_0 = 0, \tag{24}$$

where we have set the index of the series to explicitly show the frequency components of the motion due to each longitudinal harmonic. Note that this force is independent of the rf frequency, but has some phase dependence. The two terms a_{n-1} and a_{n+1} at a given spatial harmonic n are due to the backward wave at the lower harmonic $n-1$ and the forward wave at the higher harmonic $n+1$. The phase dependence of the ponderomotive force is due to the interference of the forward and backward waves at each spatial harmonic.

We can now define an average focusing strength due to the rf wave as

$$K_r = -\frac{\overline{F_r}}{r\gamma\beta^2 mc^2} = \frac{1}{8}\left[\frac{eE_0}{\beta\gamma mc^2}\right]^2 \sum_{n=1}^{\infty} \{a_{n-1}^2 + a_{n+1}^2 + 2a_{n-1}a_{n+1}[\cos(2\Delta\phi)]\} \ . \tag{25}$$

This focusing strength is of course equal to the focusing strength in the Cartesian coordinates x and y due to the linearity and symmetry of the field. For a rf wave containing only the fundamental spatial harmonic, the sum in Eq. (25) gives unity, and the phase dependence vanishes, since the forward wave does not contribute to the focusing and therefore cannot interfere with the backward wave which provides all the focusing in this case. Since the fundamental is the only spatial harmonic which is in resonance with the accelerating electrons, one usually optimizes to make all other harmonics quite small, as they dissipate power in the cavity walls, yet cannot transfer energy to the beam. Thus for most commonly encountered structures one can take the sum in Eq. (25) to be nearly equal to one. On the other hand, for an unoptimized structure, the sum can be larger. As an example, consider a square wave, which using our present normalization has the Fourier decomposition

$$E_z = E_0 \sin(\omega t+\phi_0) \sum_{n=1}^{\infty} \frac{1}{n}\cos(nkz) \ . \tag{26}$$

In this case, for $\Delta\phi=0$, the sum in Eq. (25) is equal to $(\pi^2/3)-1 \simeq 2.28$, and the focusing is quite a bit stronger than for a single harmonic.

At this point we must examine the assumptions leading to Eq. (10) to see where the approximations break down. First, one must see whether twice the rf wave number is much larger than the betatron wave number. This can be quantified by writing

$$2k \gg k_\beta = \sqrt{K_r} \simeq \frac{eE_0}{\sqrt{8}p_z c} \ . \tag{27}$$

For typical rf wave numbers $k \simeq 60 m^{-1}$ ($f=\omega/2\pi=3$ GHz), this means that the approximation we have used is valid if $p_z c \gg eE_0/4\sqrt{2}k$, or for $eE_0 = 100$ MeV/m, as in the UCLA [4] and the Brookhaven [5] rf photocathode guns, $p_z c \gg 1$ MeV.

The other approximation we have made in our derivation is in assuming that the effective mass γm is constant over a period of oscillation. Thus one expects that if $\Delta\gamma/\gamma$ is not much less than unity over one rf cell ($L_c = \pi/k$), then our results may be in error. This can be quantified as follows:

$$\frac{\Delta\gamma}{\gamma} \simeq \frac{eE_0\pi}{2\beta\gamma m_e c^2 k} \ll 1 \ , \tag{28}$$

or equivalently $p_z c \gg eE_0\pi/2k$, which is very similar to, but slightly stronger than, the inequality obtained from comparison of wave numbers. In addition, a related approximation in the derivation of Eq. (25) is that the particle velocity is nearly c, which is also true for $p_z c \gg 1$ MeV. One can conclude that for high-gradient standing wave accelerators in the S band that once the particles are very relativistic, the results of our analysis should be valid. This condition must also be satisfied for the derivation to hold for another reason, the assumption that $\beta \simeq 1$, which was made in obtaining Eq. (10).

IV. PONDEROMOTIVE rf FOCUSING IN TRAVELING-WAVE ACCELERATORS

Ponderomotive focusing in traveling-wave rf linear accelerators can be considered to be as a subset of the standing-wave rf field focusing, since, as we have mentioned above, a standing-wave accelerating field can be thought of as the superposition of a forward and a backward traveling-wave field. Some generalization is necessary, however, because traveling-wave linacs are not operated in the π mode, which has a vanishing group velocity. A traveling (forward) wave linac is generally operated in a mode characterized by a phase shift per period of $p\pi/q$, where p and q are integers.

The accelerating field in such a traveling-wave structure, with spatial periodicity over a length d, can be specified by the following Floquet form [6]:

$$E_z = E_0 \sum_{n=-\infty}^{\infty} b_n \exp[i(\omega t - \beta_n z)] \ , \tag{29}$$

where now $E_0 \equiv \overline{E_{\rm acc}}$, and $\beta_n = \beta_0 + (2\pi n/d)$. The constant $\beta_0 d = \psi$ is the phase shift per period. For relativistic electrons $\beta_0 = \omega/c = k$, and b_0 is unity in this normalization.

Now, setting $\omega t = \beta_0 z$ for maximal acceleration (suppressing an inconsequential constant phase), we can write

$$F_r = 2E_0 kr \sum_{n=-\infty}^{\infty} \frac{inb_n q}{p} \exp\left[i \left[\frac{2knzq}{p} \right] \right] . \tag{30}$$

Note that since there is not a large-amplitude backward wave, as is found in the standing-wave case, the ponderomotive force should be much smaller in a traveling-wave field. Using Eq. (10) (averaging over a period of the *structure*), we can write

$$\overline{F_r} = -r \frac{(2eE_0)^2}{\gamma m_e c^2} \sum_{n=-\infty}^{\infty} |b_n|^2 \ , \quad n \neq 0 \ . \tag{31}$$

The focusing strength associated with this force is

$$\overline{K_r} = \left[\frac{2eE_0}{\beta\gamma m_e c^2} \right]^2 \sum_{-\infty}^{\infty} |b_n|^2 \ , \quad n \neq 0 \ . \tag{32}$$

It is apparent that the backward wave that is "missing" here contributed much of the focusing in the standing-wave case, as the focusing is much diminished in traveling-wave linacs. In the limit that there is only the fundamental spatial harmonic ($n=0$), then the focusing disappears completely. Given that traveling-wave linac structures, like standing-wave structures, are optimized to make spatial harmonics other than the fundamental much smaller than unity, the ponderomotive force in traveling-wave linacs should be much smaller than that found in traveling-wave structures. It is interesting to note that a particle traversing a traveling-wave structure in a direction opposite to the accelerating wave will undergo ponderomotive focusing equivalent to that found in a standing-wave structure. This effect has been experimentally observed [7].

V. PONDEROMOTIVE FOCUSING AND ELECTROMAGNETIC ENERGY DENSITY

As an alternative to the examination of the microscopic dynamics of the electrons during the fast oscillation which, when averaged, gives a ponderomotive force, we now explore the relationship between the ponderomotive focusing and the average variations in the electromagnetic energy density [8]. This line of thought is of course inspired by the E_0^2 dependence of the ponderomotive force. For the standing-wave case, keeping only the fundamental spatial harmonic, the average (over one cell) of the electromagnetic energy density is

$$\overline{U} = \frac{\epsilon_0}{2}(\overline{\mathbf{E}^2} + c^2\overline{\mathbf{B}^2}) \tag{33}$$

$$= \frac{\epsilon_0}{2} E_0^2 \left[1 + \frac{(kr)^2}{4} \right] . \tag{34}$$

The transverse gradient of this energy density gives a net inward electromagnetic pressure gradient (force per unit volume) of

$$-\frac{\partial \overline{U}}{\partial r} = -r\frac{\epsilon_0}{4}(kE_0)^2 \ . \tag{35}$$

In order to convert this quantity to the correct ponderomotive force [as given by Eq. (25)], one must assign an effective volume of the interaction between the electron and the electromagnetic field of

$$V_{\rm eff} = \frac{2\pi r_e}{\gamma k^2} \ . \tag{36}$$

In this way, the ponderomotive force on an electron can be envisaged as being due to the gradient of the field density, with the charged particle tending to move towards the minimum in the electromagnetic energy density. This effective volume can be qualitatively justified as follows: the length r_e/γ can be thought of as the Lorentz contracted electromagnetic length of the electron, and the factor $2\pi/k^2$ due to the inherent cross-sectional size of the classical electromagnetic rf wave (or, equivalently, the quantum-mechanical photons which make up the classical field).

The microscopic description of the motion gives some further insight into the physical mechanism by which the particle seeks the minimum in electromagnetic energy density. The fast oscillatory motion allows the particle to "test" the surrounding regions of space. The field in the regions with larger energy density will tend to repulse the beam because of the phase relationship between the force and the motion (they are π out of phase), as is shown in Eq. (5).

VI. THE ENVELOPE EQUATION

The strong ponderomotive rf focusing term derived for standing-wave linacs [Eq. (25)] can be included in the generalized paraxial rms beam envelope equation for a cylindrically symmetric system as derived by Lawson [9], which explicitly includes the effects of acceleration in a straightforward manner. We obtain

$$A''+\left[\frac{\gamma'^2(\gamma^2+2)}{4\beta^4\gamma^4}+K_r\right]A-\frac{\beta\gamma Q}{A}-\frac{\epsilon_n^2}{A^3}=0\ , \quad (37)$$

where $\gamma'=\overline{eE_{\rm acc}}/mc^2\simeq eE_0/2mc^2$ for an optimized π-mode standing-wave accelerating structure with $v_b=c$, the normalized rms beam size $A=\sigma_r\sqrt{\beta\gamma}$, $\epsilon_n=\beta\gamma\epsilon$ is the normalized rms emittance, and $Q=I/\beta^3\gamma^3 I_0$ is the perveance of the beam, with $I_0=ec/r_e\simeq 17$ kA for electrons.

If $\Delta\gamma/\gamma$ changes by much less than unity over a rf cell, we obtain an approximate equilibrium solution ($A''=0$) to the envelope equation, giving a constant rms beam radius

$$\sigma_r\simeq\left[\frac{2\epsilon_n mc^2}{eE_0}\right]^{1/2}=\left[\frac{\epsilon_n}{\gamma'}\right]^{1/2}, \quad (38)$$

which is *independent* of energy. For emittances and accelerating gradients typical of proposed superconducting standing-wave linear colliders [10] ($\epsilon_n\simeq 10^{-6}$ m rad, $\gamma'\geq 60$ m^{-1}), this gives an equilibrium beam size of $\sigma_r\simeq 130$ μm, which is nearly the same as present designs would indicate due to external focusing at the beginning of the linac.

As an example of an implementation of this envelope equation description of the beam radius dynamics, we present a comparison of the results of a PARMELA (a multiparticle code which models beam propagation under the influence of rf forces, external magnetic fields, and self-consistent space-charge forces) simulation of the rms beam envelope with solutions to the envelope equation. The case we show in Fig. 1 is a 8.568-GHz 12.5-cell standing-wave rf photocathode gun with a Brookhaven-style cavity shape, which is currently being designed at UCLA. The accelerating field $E_0=225$ MV/m, the beam length $\sigma_z=0.15$ mm, and the charge in the bunch is 1 nC. Strong space-charge effects on the beam envelope are apparent in the simulation, as the beam expands rapidly in the first few cells. One can also see in the simulation the oscillations of the beam envelope as it encounters the periodically varying rf focusing forces, which give rise to the ponderomotive focusing we have analyzed here. For comparison with the simulation, we numerically solve the envelope equation (37), choosing the much less ambiguous rms beam size and its derivative at the end of the accelerating structure as the initial conditions, and integrate backwards towards the photocathode (the envelope equation is invariant under time reversal). The agreement with the simulation is remarkable, all the way until the first few cells, where the momentum is below 5 MeV/c, as we might expect from our discussion of the limitations of our derivation. Also, for comparison, we integrate Eq. (37), omitting the rf focusing term, using the final conditions of the previous solution as the new initial conditions for a forward integration. The beam envelope diverges to a radius three times as large as the simulated radius at the end of the accelerating structure, indicating the importance of the rf focusing term.

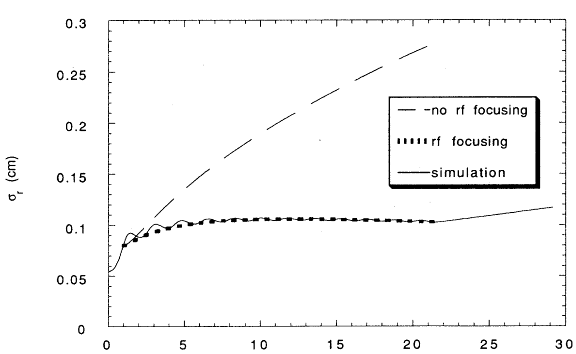

Figure 1. Comparison of PARMELA simulation to envelope equation results, including and omitting rf focusing. Parameters of gun: σ_z=.15 mm,1 nC of charge, E_0=225 MV/m.

FIG. 1. Comparison of PARMELA simulation to envelope equation results, including and omitting rf focusing. Parameters of gun: $\sigma_z=0.15$ mm, 1 nC of charge, $E_0=225$ MeV/m.

For completeness, we write here the extension of the envelope equation in Cartesian coordinates for nonaxisymmetric beams in standing-wave rf linacs, without external solenoid or quadrupole focusing. Terms corresponding to introduction of these effects can easily be incorporated [9]. In the horizontal dimension we have

$$X''+\left[\frac{\gamma'^2(\gamma 2+2)}{4\beta^4\gamma 4}+K_r\right]X-\frac{2\beta\gamma Q}{X+Y}-\frac{\epsilon_{nx}^2}{X^3}=0\ , \quad (39)$$

and in the vertical dimension

$$Y''+\left[\frac{\gamma'^2(\gamma^2+2)}{4\beta^4\gamma 4}+K_r\right]Y-\frac{2\beta\gamma Q}{X+Y}-\frac{\epsilon_{ny}^2}{Y^3}=0\ . \quad (40)$$

Here the normalized rms beam sizes are $X(Y)=\sigma_{x(y)}\sqrt{\beta\gamma}$, and the normalized emittances are $\epsilon_{nx(y)}=\beta\gamma\epsilon_{x(y)}$.

VII. rf FOCUSING OF POSITRONS AND PROTONS

Since positrons possess the same mass as electrons, and undergo the same forces while accelerating in a linac, the results of this analysis may be applied without modification to positrons. An obvious application is in understanding the transverse dynamics of positron beams in standing-wave linear colliders, a subject addressed further below.

Protons (or equivalently H^- ions), while having large mass, have small momenta in standing-wave rf linacs, generally under 1 GeV/c (at higher momenta it is more efficient to accelerate protons in a synchrotron) and thus one must consider the effects of the transverse rf fields on the motion of the protons. On the other hand, the accelerating gradients in proton linacs are generally low, since the frequency is generally lower than in electron linacs. This will of course cause the ponderomotive force to be smaller in proton linacs. In addition, modifications must be made in the derivation of the time-averaged transverse force to more massive particles (cf. Ref. [11]). The change in particle velocity over the length of a cell is an important effect in proton linacs. This change in velocity means less time is spent by a proton in the second half of a cell than the first, which causes a lack of cancellation of the transverse forces over the complete cell.

In addition, because the velocities are not close to c there is a strong phase-dependent transverse effect arising from the fact that the protons are nonrelativistic and do not feel strong effects of the magnetic field of the rf wave. In the frame of the rf wave, the synchronous wave's field is electrostatic, and for a bunch phase chosen to give phase (longitudinal) stability $\partial E_z/\partial z < 0$, which by symmetry and Laplace's equation requires $\partial E_x/\partial x = \partial E_y/\partial y > 0$, and the synchronous forward wave's transverse electric field is actually defocusing. The nonsynchronous components in general do not overcome this effect unless the synchronous phase of the rf is alternated along the linac structure, giving a class of ponderomotive focusing known as alternating phase focusing [11]. These considerations, combined with the relatively low accelerating gradients found in heavy-particle linacs, will cause the ponderomotive rf focusing to be a less important effect in these devices.

VIII. PRACTICAL IMPLICATIONS

It is interesting to observe that the focusing strength given by Eq. (25) has the same momentum dependence as that derived from solenoidal [9] focusing,

$$K_{r,\text{sol}} = \left[\frac{eB_z}{2\beta\gamma m_e c} \right]^2 . \tag{41}$$

Comparing this to Eq. (25), we can see the ponderomotive force due to the transverse rf fields in a standing-wave linac are equivalent to those of an applied solenoidal field of effective strength

$$B_{\text{eff}} \simeq \frac{E_0}{\sqrt{2}c} . \tag{42}$$

For the accelerating field amplitude used in our rf photocathode source example, $E_0 = 225$ MV/m, and $B_{\text{eff}} \simeq 5.3$ kG, which is a very strong solenoid (to practically obtain a dc field much stronger than this probably requires use of a superconducting magnet). If one has the first accelerator section attached directly to the rf photocathode to provide a long focusing length, the ponderomotive force may make external solenoidal focusing superfluous. This is the scenario illustrated by Fig. 1. In this case a strongly diverging beam with non-negligible space-charge defocusing has been controlled only through the naturally occurring ponderomotive rf focusing. The marriage of the rf photocathode source and the major part of the linac would therefore seem to be a beneficial design philosophy for high-brightness electron sources.

The existence of strong ponderomotive focusing due to the rf cavities also has some implications for linear collider design, especially linear colliders based on high-gradient standing-wave superconducting structures. In addition to modifying the basic focal properties of the beam line, if optical elements are misaligned from the design axis, the beam will also be steered. Chromatic effects in the subsequent particle trajectories may dilute the transverse phase space, either directly or through enhanced wake-field effects (beam breakup), thus increasing the small emittances needed for achieving high luminosity collisions. Schemes which correct these effects [12,13] may be stressed by the fact that the misalignable cavities are not localized, but in fact occupy most of the beam line. These potential problems may be aggravated by the relative difficulty in aligning superconducting cavities inside large cryostats. These effects should, however, be relevant only at the low-energy end of the collider, since the rf focusing strength scales as γ^{-2}, while conventional quadrupole focusing strength scales as γ^{-1} [14,15].

IX. CONCLUSIONS

The physical effects explored in this paper are of most interest in describing beam particle trajectories and beam envelopes in high-gradient standing-wave electron (or positron) linacs. The implications of these effects may be either positive—rf focusing mitigates the necessity for an external focusing system—or negative—a superconducting linear collider may have tighter alignment tolerances for cavity placement. In any case, the arrival of higher-gradient standing wave structures, driven by rf photocathode source development, as well as interest in high-gradient superconducting linacs, means that these effects are or will be non-negligible in many present and future linear accelerators. We have presented here a description of these phenomena and a framework for obtaining the beam envelope when the rf focusing is important. More analysis for rf waves without axisymmetry (e.g., rf quadrupole, or higher, modes), or extensions to include nonlinear radial dependences of the fields, is of course necessary to more completely describe the transverse dynamics of beams under the influence of rf fields in high-gradient linear accelerators.

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